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A NEW APPROACH TO THE DESIGN OF TIME-VARYING CONTROL SYSTEMS WITH APPLICATION TO THE SPACE SHUTTLE BOOST

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May 1973

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E-2765

A NEW APPROACH TO THE DESIGN OF TIME-VARYING CONTROL SYSTEMS WITH APPLICATION TO THE SPACE SHUTTLE BOOST

Abstract

An investigation is made of a general approach toward the analysis and design of closed loop control for slowly time-varying linear systems. The approach is by the method of generalized multiple scales, in which slow and fast dynamics are systematically separated by employing different "clocks" which measure time at varying rates. The clocks, which are necessarily nonlinear functions of time, are ~~chosen~~ appropriately such that the system dynamics are asymptotically invariant with respect to the new time scale. A transfer function relating the output to the input for general linear slowly time-varying systems is developed and represents the actual system under certain conditions. The clock functions is shown to satisfy an algebraic characteristic equation and can be determined in general in terms of the coefficients. The time-invariant case arises as a natural limit and a special case of our general approach. Control system design is carried out with respect to the transformed, approximate system representation using standard synthesis techniques. Transformation back to real time results in time-varying control. We have thus provided a useful framework for further analysis of properties such as stability, parameter sensitivity and response of time-varying control systems. The approach can be viewed as an extension of time-invariant linear feedback control theory. Our approach is valid continuously through the time variation and is a substantial improvement over the "frozen" approximation (a special case of our approach), which is limited to very short time intervals. The method is applied to the control design of the space shuttle during the initial boost phase. A preliminary design of the control is presented both for the second order approximation and the third order representation of the shuttle dynamics. For the latter, conditions for minimum lateral drift are determined from the multiple scales formalism. Feedback gains are determined for minimum drift for two cases including feedback of angle of attack, pitch and pitch rate.

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1. Introduction

Analysis of the dynamics of linear time-varying (t-v) systems has been the object of considerable study and research. However, the problem is not easy and motivates attempts to develop new methods of analysis. Further, the problem of controlling t-v systems is intimately related to our ability to predict, at least qualitatively, the dynamic evolution of general t-v systems. A natural extension of such an understanding is the development of synthesis and design techniques for t-v systems. It has been shown that the method of generalized multiple scales[1, 2] enables us to develop an analytical description of the response of a general class of t-v systems. In this paper an investigation is made of developing analysis and control system synthesis techniques for feedback control of t-v systems by using the concept of generalized multiple scales. By this means, rapid and slow motions of the system dynamics are systematically separated, leading to an asymptotic description of the dynamics of general, slowly time-varying systems. We are, therefore, able to analyze the closed loop dynamics of t-v systems by an extension of classical techniques of analysis of constant linear control systems. This approach is then applied to the analysis and control of the space shuttle vehicle from launch through the initial boost phase (up to staging).

The origin of the method of multiple scales can be traced to the works of Krylov, Bogoliubov and Mitropolsky[3] who allowed the constants arising in direct perturbation theory to be slowly varying functions. The method in its early form was developed in the context of irreversible processes in statistical mechanics by Frieman and Sandri[4] and of some nonlinear differential equations by Cole and Kevorkian[5]. These applications considered linear scales—ie the time scales were linear functions of the original independent variable. The method was generalized by Ramnath[1, 2] and Sandri[2] to include general scales which could be nonlinear functions as well as complex quantities.

2. The Method of Multiple Scales

We will now briefly discuss the concept of multiple scales and its application to the analysis of t-v systems in general. The discussion is of necessity brief here and proceeds from a control engineer's point of view. For a more complete presentation of the technique the reader is referred to references 1 and 2.

We will now consider the possibility of reducing linear time-varying (t-v) systems to time-invariant systems. Such an exact conversion in the general case is impossible inasmuch as it is tantamount to solving t-v equations exactly in terms of elementary transcendental functions, which is known to be impossible[1]. An important aspect of constant linear systems is that exact solutions can be expressed in terms of exponentials of arguments which are linear in the independent variable.

The solution of time-varying systems involve higher transcendental functions such as hypergeometric functions and Mathieu functions which cannot be represented in terms of exponentials. Nonetheless, we seek to develop approximate solutions to t-v systems in terms of elementary transcendental functions. The best approach to approximations is by means of asymptotic theory. For purposes of engineering analysis the difference between asymptotic approximations and mathematically exact solutions is only academic.

The method of multiple scales is particularly useful when a phenomenon exhibits a mixture of fast and slow motions. These are separated in a systematic manner by employing a number of independent observers who perform readings using clocks which count time at different rates, changing continuously with respect to real time. Mathematically time, t, is extended into space of larger dimension, ie into a vector, $\vec{\tau}$; with components τ_i, $i = 1, \ldots n$ depending on t. More generally,

$$\tau_i = \tau_i (t, \epsilon) \tag{2.1}$$

where ϵ is a small, positive parameter, being a measure of the separation of the slow and fast motions. Let the system dynamics be described by a class of single-valued mappings $\{\phi\}$ with domain δ and range ρ. The nature of the mappings, domain and range are assumed to be known. Thus

$$\phi : \delta \rightarrow \rho \tag{2.2}$$

The domain δ is embedded in a set $\underline{\delta}$ of higher dimension, or dim $\underline{\delta} \geqq \delta$. Now $\underline{\delta}$ is called an extension of the domain. The single-valued mapping associated with the extension of δ is denoted by E_δ. Now $\underline{\phi}$, the extension of ϕ, is defined as a single-valued mapping of $\underline{\delta}$ into $\underline{\rho}$ with $\underline{\rho} \supset \rho$, if and only if

$$\underline{\phi} \cdot E_\delta = \phi \tag{2.3}$$

where $\underline{\phi} \cdot E_\delta$ denotes the composition of two mappings E_δ and $\underline{\phi}$. This is shown in Fig. 1a.

An accurate representation of extension is given by the commutative sequence (Fig. 1b). More specifically, let us consider the dynamics of a time-varying system to be described by an equation

$$\mathcal{L}\{y (t, \epsilon), x(t, \epsilon) t, \epsilon\} = 0 \tag{2.4}$$

where $\mathcal{L}$ is a linear operator, y is the system response x is the input and ϵ is a small parameter. We extend

$$t \rightarrow \{\vec{\tau}\} \tag{2.5}$$

where $\vec{\tau}$ is a vector of n dimensions. In general $\tau_i = \tau_i (t, \epsilon)$, $i = 1, 2, \ldots n$. Now

$$y (t, \epsilon) \to \bar{y} (\vec{\tau}, \epsilon) \tag{2.6}$$

In general we could make an asymptotic expansion of $\bar{y}$ in powers of ϵ. For our present purposes we will consider only $\bar{y}(\tau)$. We choose $\tau_i (t, \epsilon)$ such that the functional dependence of $\bar{y}$ on τ_1 becomes particularly simple. Thus we arrive at the natural variables to describe the solution. After a suitable choice of τ_i we will restrict the extended solution $\bar{y}(\vec{\tau})$ along the "trajectories" $\tau_i(\tau, \epsilon)$ to obtain the solutions in real time, t. We must note that in order to represent linear t-v systems in terms of constant linear systems, it is essential to distort the independent variable, t, in a nonlinear fashion.

3. Application to Time-Varying Systems

We will now develop an analytical description of a general class of linear time-varying systems by using the above concepts. The general theory has been presented in Ref. 1, 2 and we will only consider the problem from the standpoint of control. We will develop the theory for general linear t-v systems of the n^{th} order. Specialization to systems of low order is straight forward. We will consider system description in the scalar form although the analysis can be extended to the vector form as well.

Consider an n^{th} order linear time-varying system described by;

$$\sum_{i=0}^{n} \omega_i \, y^{(i)} = K \sum_{j=0}^{m} \alpha_j \, x^{(j)} \tag{3.1}$$

where: $m \leqq n$; ω_i, α_i are slowly varying functions of time, K is a constant,

$$y^{(i)} (t) \triangleq \frac{d^i y}{dt^i}$$

and

$$x^{(j)} (t) \triangleq \frac{d^j x}{dt^j} \quad .$$

The coefficients ω_i can therefore be considered to be dependent on the variable $\tilde{t} = \epsilon t$ where $0 < \epsilon << 1$. Equation (3.1) can be reparameterized and written in the form

$$\sum_{i=0}^{n} \epsilon^i \, \omega_i \, (\tilde{t}) \, y^{(i)} \, (\tilde{t}) = K \sum_{j=0}^{m} \epsilon^j \, \alpha_j \, (\tilde{t}) \, x^{(j)} \, (\tilde{t}) \tag{3.2}$$

In order to develop an analytical asymptotic approximation to the system response, we extend the variables as follows.

$$\tilde{t} \rightarrow \{\vec{\tau}\} \qquad \text{(a)}$$

and

$$y\,(\tilde{t},\ \epsilon) \rightarrow \bar{y}\,(\vec{\tau}) \qquad \text{(b)} \qquad (3.3)$$

$$x\,(\tilde{t}) \rightarrow \bar{x}\,(\vec{\tau}) \qquad \text{(c)}$$

specifically we consider the time vector $\vec{\tau}$ to be of two dimensions with components τ_o and τ_1. We choose

$$\tau_o = t,\ \tau_1 = \int \frac{k(t)}{\epsilon}\, dt \qquad (3.4)$$

where k(t) is a "clock" function, as yet undetermined. The system differential equation, in the leading order in ϵ, is written as:[1,2]

$$\sum_{i=0}^{n} \omega_i\,(\tau_o)\, k^i \frac{\partial^i \bar{y}}{\partial \tau_1{}^i} = K \sum_{j=0}^{m} \alpha_j\,(\tau_o)\, k^j \frac{\partial^j \bar{x}}{\partial \tau_1{}^j} \qquad (3.5)$$

$$\bar{y} \sim \bar{y}\,(\tau_1)$$

On Laplace transformation with respect to τ_1 and rearranging, the above equation can be written as:

$$\frac{y}{x} = G\,(\xi,\ \tau_o) \qquad (3.6)$$

where

$$G\,(\xi,\ \tau_o) = \frac{K \sum_{j=0}^{m} \alpha_j\, \xi^j}{\sum_{i=0}^{n} \omega_i\, \xi^i} \qquad (3.7)$$

and ξ = sk; s is the Laplace variable. We now have the input-output relation as a transfer function G with poles and zeros moving slowly. The variable ξ, which is not restricted to be real, defines the Laplace-clock space in which the system differential equation can be asymptotically represented by an algebraic expression. The singularities of this expression are significant in developing a solution in real time t. Their importance is manifest in inverse Laplace transforming to the time domain, which is done in conjunction with the restriction of the extended solutions.

Given $\xi(\tau_o)$, and the poles of $G(\xi, \tau_o)$ the fundamental solutions of (3.5) are given by:

$$\bar{y} = \sum_{i=1}^{n} C_i(\tau_o) \exp(\tau_{1i}) \tag{3.8}$$

Upon restriction the solution is written as:

$$y(\tilde{t}, \epsilon) = \sum_{i=1}^{n} C_i(\tilde{t}) \exp\left(\int \xi_i(t)\, dt\right) \tag{3.9}$$

where C_i are coefficients suitably determined in a partial fraction expansion of G.

We must note that (3.9) represents the dominant part of the exact solution. Accuracy of the solution can be improved by considering slower variations (ie in τ_o) as well. To leading order C_i are considered to be constants.

The advantage of the above approach is that we are able to analyze a general class of linear time-varying systems by an extension of classical theory. We have developed a transfer function of a t-v system leading to suitable block diagram representation, root locus analysis for the closed loop, design of compensation networks and so on. In the present analysis the poles are assumed to be distinct.

4. Application to the Boost Phase of the Space Shuttle

In this section we will discuss an approach towards application of the above theory to the preliminary design of the control system for the space shuttle vehicle from launch during the initial boost phase. The system parameters such as mass, inertia, velocity vary continuously through boost and the dynamics are, therefore, time-varying. Techniques of linear, constant-coefficient systems analysis are not applicable. Because of this fundamental difficulty, previous approaches have designed control systems by approximating the t-v dyanmics by linear constant systems on the basis of "freezing" the system at a number of instants of time.
The validity and usefulness of such an approach is very limited and could lead to erroneous conclusions in regard to system stability. The system must be frozen at a large number of time instants and at best, this approach has a very short range of validity. There is, therefore, a need for a better and more accurate representation of the system dynamics. The multiple scales theory offers precisely such an approach. By means of this theory we are able to accurately represent the system dynamics uniformly through the time variation, in terms of simply calculable functions. It eliminates the need for several control system analyses (each at a different instant of time) by the frozen method.

We consider motions of the shuttle vehicle about a launch trajectory. The system is represented by a simple mathematical model mainly to illustrate analysis and design by the new technique. The vehicle is considered to be a rigid body and fuel sloshing is neglected. With the usual notation, the motion of the vehicle is described by [6];

$$\ddot{z} = -\left[\frac{(T_T - D)}{m} + g \cos \theta_o\right] \theta - \frac{L_\alpha}{m} \alpha + \frac{T_c}{m} \delta \qquad \text{(a)}$$

$$\ddot{\theta} = \mu_\alpha \alpha + \mu_c \delta \qquad \text{(b)}$$

$$\alpha = \theta + \frac{\dot{z}}{V} + \alpha_w \qquad \text{(c)} \qquad (4.1)$$

$$\text{where } \mu_\alpha = L_\alpha \ell_\alpha / I \text{ and } \mu_c = T_c \ell_c / I \qquad \text{(d)}$$

The coefficients of these differential equations vary slowly during the boost phase in a manner that depends on the actual trajectory. The variations of the coefficients for a specific trajectory are given in Table I. In accordance with the multiple scales theory in §3, the dynamics of (4.1) are dominantly described by:

$$\begin{bmatrix} \xi^2 & (T_T - D)/m + g \cos \theta_o & L_\alpha/m \\ 0 & \xi^2 & -\mu_\alpha \\ -\xi/V & -1 & 1 \end{bmatrix} \begin{bmatrix} z \\ \theta \\ \alpha \end{bmatrix} = \begin{bmatrix} T_c/m \\ \mu_c \\ 0 \end{bmatrix} \delta + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \alpha_w \qquad (4.2)$$

or in vector form

$$AX = C\delta + W\alpha_w \qquad (4.3)$$

where the terms in the vector-matrix equation (4.3) are defined by comparison to (4.2) and ξ is the Laplace-clock variable ($\xi = sk$). As the coefficients vary only on the slow scale τ_o, (4.1) can be studied as a constant coefficient system (4.2) with respect to τ_1, the fast scale. The response of (4.1) to a control input δ or a wind input α_w is described fundamentally by the set (4.2) through the nonlinear clock $k(\tau_o)$. For any choice of $k(\tau_o)$ we have a scale τ_1 and a solution $\tilde{x}(\tau_1)$. However, the optimal choice of k(t) is that which renders τ_o and τ_1 independent, which is the implicit rationale in the multiple scales approach. This criterion leads to the optimal choice of the clock function k(t) to be given by the eigenvalues of A. The solution is then expressible in the asymptotic form,

Table I

Values of Parameters

The following functional forms for the parameters of the shuttle 049 vehicle after consulting Ref. (7) have been used.

$$\ell_\alpha(t) = -(5.17 + 0.0417t) \text{ ft}$$

$$\ell_c(t) = (77.08 + 0.0417t) \text{ ft}$$

$$I_y(t) = (323 - 1.55t)\, 10^6 \text{ slug ft}^2$$

$$m(t) = 1.712 \times 10^5 - 815t \text{ slugs}$$

$$S = S_{REF} = 3420 \text{ ft}^2$$

$$g = 32.2$$

$$\theta_o = 0.382 \text{ radian} = 2.398 \text{ deg}$$

$$C_{L_\alpha} = 3.21 \text{ lbs/radian}$$

$$C_D = 0.2 \text{ lb/radian}$$

$$T_c^* = 47 \times 10^4 \text{ lbs}$$

$$T_c = \text{Effective control thrust for pitch control}$$

$$= \left(2 + \frac{355}{255}\right) T_c^* = 5.92 \times 10^5 \text{ lbs}$$

$$q(t) = t^2 \exp\left[-1.7235804 + 9.7594075 \times 10^{-3}\, t - 2.7600722 \times 10^{-4}\, t^2\right]$$

$$V(t) = (5.4621582 + 0.1789339t)\, t$$

$$T_T = T_{TOTAL} = (893 - 1.428t)\, 10^4 \text{ lb}$$

$$D = \text{DRAG} = 684\, q \text{ lbs}$$

$$L_\alpha(t) = C_{L_\alpha} S\, q = 10980\, q \text{ lb/radian}$$

$$\mu_\alpha(t) = \ell_\alpha L_\alpha / I_y \; ; \; \mu_c(t) = \ell_c t_c / I_y$$

$$x(t) \sim x_s(\tau_o)\,\tilde{x}(\tau_1) \tag{4.4}$$

where $\tilde{x}(\tau_1)$ is the fast (or dominant) part and $x_s(\tau_o)$ is the slow part and τ_o, τ_1 are as defined in (3.4). The response of θ to an input δ is described by the "transfer function"

$$\frac{\theta}{\delta} = \frac{\mu_c\left[\xi + \frac{L_\alpha}{mV}\left(1 + \frac{\ell_\alpha}{\ell_c}\right)\right]}{\xi^3 + \frac{L_\alpha}{mV}\xi^2 - \mu_\alpha\xi + \left(\frac{T_T - D}{m} + g\cos\theta_o\right)\left(\mu_\alpha/V\right)} \tag{4.5}$$

The open loop dynamics are altered by feedback as follows. Let the control law be

$$\delta = -K_A\left[K_\theta\,\theta + K_R\,\dot{\theta} + K_\alpha\,\alpha\right] \tag{4.6}$$

The equations of motion can now be written as:

$$\begin{bmatrix} \xi^2 & \left(\frac{(T_T - D)}{m} + g\cos\theta_o + K_A\frac{T_c}{m}(K_\theta + K_R\,\xi)\right) & \left(\frac{L_\alpha}{m} + K_A K_\alpha\frac{T_c}{m}\right) \\ 0 & \left(\xi^2 + \mu_c K_A (K_\theta + K_R\,\xi)\right) & \left(-\mu_\alpha + \mu_c K_A K_\alpha\right) \\ -\frac{\xi}{V} & -1 & 1 \end{bmatrix}\begin{bmatrix} z \\ \theta \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\alpha_w \tag{4.7}$$

ie

$$B\,X = W\,\alpha_w \tag{4.8}$$

The basic modes of motion are given by det B = 0, ie

$$\Delta \equiv \xi^3 + B_2\,\xi^2 + B_1\,\xi + B_o = 0 \tag{4.9}$$

where:

$$B_2 = \mu_c K_A K_R + \left(\frac{L_\alpha + K_A K_\alpha T_c}{m\,V}\right) \tag{4.10a}$$

$$B_1 = \mu_c K_A (K_\theta + K_\alpha) - \mu_\alpha + \frac{K_A K_R T_c}{m\,V}\left(\mu_\alpha + \frac{\mu_c L_\alpha}{T_c}\right) \tag{4.10b}$$

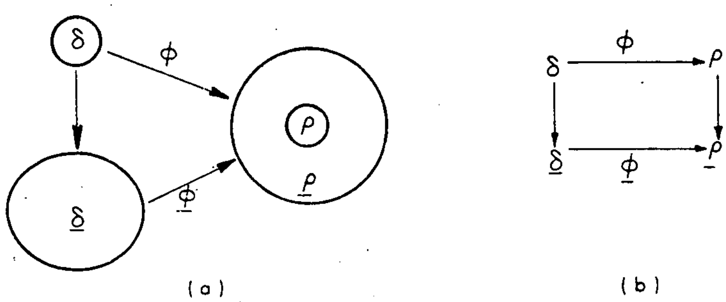

Fig. 1. Composition Of Mappings In Extension.

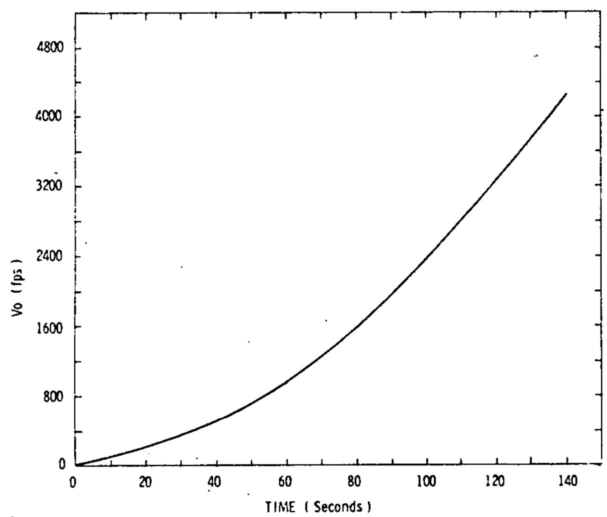

Fig. 2. Velocity Time History.

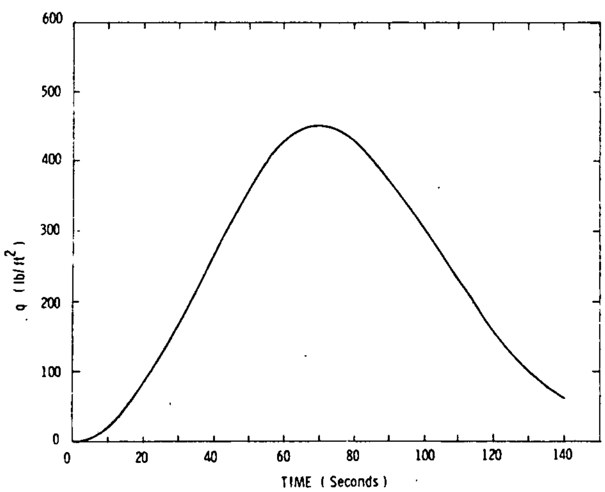

Fig. 3. Dynamic Pressure Variation.

$$B_o = \frac{K_A K_\theta T_c}{m V}\left(\mu_\alpha + \frac{\mu_c L_\alpha}{T_c}\right) - \left(\frac{T_T - D}{m} + g \cos \theta_o\right)\left(\frac{\mu_c K_A K_\alpha - \mu_\alpha}{V}\right)$$

(4.10c)

In order to develop solutions for the SSV 049 vehicle motions, analytical expressions were derived to describe the vehicle parameter variations. The representations are best in a least squares sense and are based on known data[7]. Some of the parameter variations are shown in Fig. 2-5. For the 049 vehicle $\ell_\alpha < 0$ (c.g. is behind the center of pressure) and the vehicle is aerodynamically stable ($\mu_\alpha < 0$).

It is interesting to study the approximations used to represent the vehicle system in earlier engineering analyses. In regions of negligible aerodynamic pressure q, (4.5) is approximated by:

$$\frac{\theta}{\delta} = \frac{\mu_c}{\xi^2} \tag{4.11}$$

When q is not negligible V is large and so (4.5) is approximated by:

$$\frac{\theta}{\delta} = \frac{\mu_c}{\xi^2 - \mu_\alpha} = G_1 \tag{4.12}$$

In these cases, one real clock function can be chosen to convert the system to a time-invariant system. It is trivial in the case of (4.11), which is a double integrator. With (4.12), substituting $\xi = sk$ suggests the choice of

$$k = (-\mu_\alpha)^{1/2} \tag{4.13}$$

which converts (4.12) to

$$\frac{\theta}{\delta} = \frac{-\mu_c/\mu_\alpha}{s^2 + 1} = G_2 \tag{4.14}$$

While Laplace transforms and root locus methods cannot be applied to t-v systems in a direct form, they are applicable in the form we have developed, ie (4.14). Root variations are eliminated in (4.14). Control system design can be carried out by conventional methods such as by root locus (Fig. 6). The clock is shown in Fig. 7. The compensation network is designed in the s- space with a pole a and a zero b, which are constants. Transformation back to real time results in a time-varying compensation in which the pole, zero and the gain vary continuously. ie,

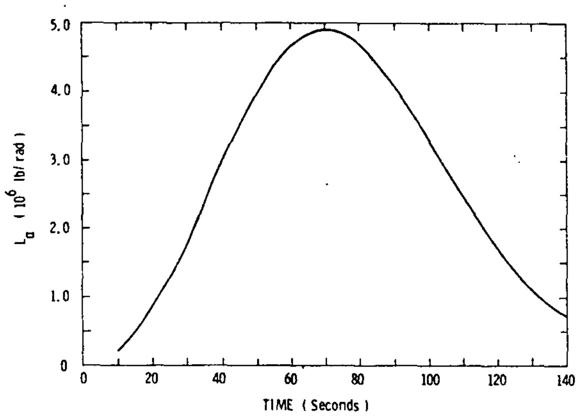

Fig. 4. Aerodynamic Load (per unit α) Variation.

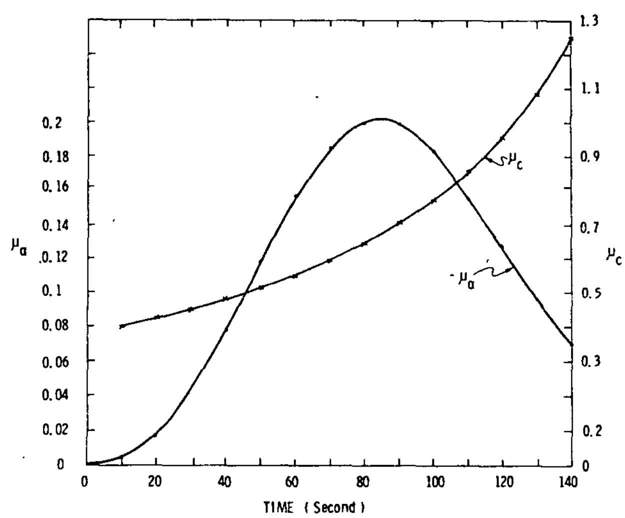

Fig. 5. Aerodynamic And Control Moment Parameters.

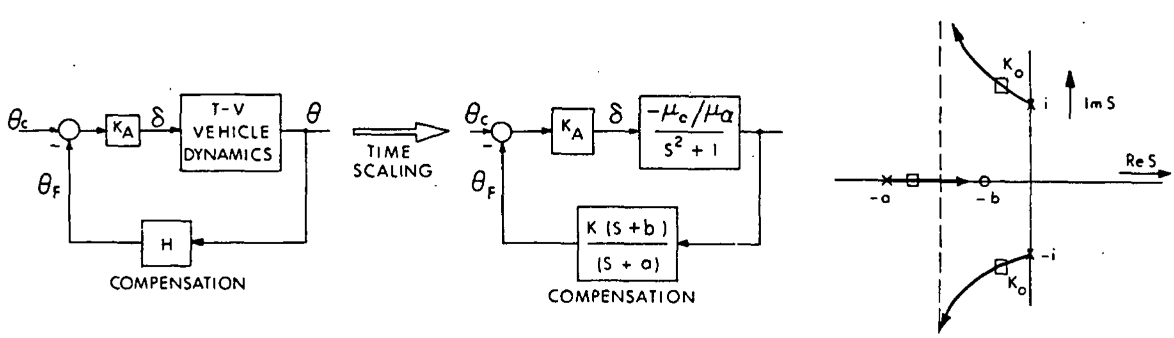

Fig. 6. Control Design In Transformed Space.

$$\frac{\theta_F}{\theta} = H = (-\mu_\alpha/\mu_c)\frac{K_o\,(s + b\sqrt{-\mu_\alpha})}{(s + a\sqrt{-\mu_\alpha})} \tag{4.15}$$

where K_o is the value of the gain corresponding to the desired location of roots on the s- plane. The results of this design for the 049 vehicle is given in Fig. 8, where $b = 0.5$, $a = 1.5$, $K_o = 1$. The neutrally stable open loop system has been stabilized. The response of the system can be expressed analytically at any time we wish.

We will now consider the third order equation (4.9) describing more general motions in the pitch plane. The root configuration for the 049 vehicle for the specific trajectory chosen is shown in Fig. 9. Feedback control gains are chosen as follows.

Minimum Drift Condition

We will now examine the response of the complete system to wind inputs. In particular we will derive the condition for minimum lateral drift in a rigorous manner through the multiple scales theory. In order to do so we will need the generalized final value theorem of Laplace transforms.

Lemma: If $f(t) \sim g(t)$ as $t \to \infty$ then

$F(s) \sim G(s)$ as $s \to 0$,

where $F(s)$, $G(s)$ are the Laplace transforms of f and g.

Using this lemma, we know from (4.4) and (4.8) and considering Laplace transforms with respect to τ_1, that:

$$\begin{aligned} x(t) &\sim \tilde{x}(\tau_1(t)) && \text{(a)} \\ x(s) &\sim \tilde{x}(s) && \text{(b)} \end{aligned} \tag{4.16}$$

The limiting value is given by:

$$\lim_{\tau_1 \to \infty} \tilde{x}(\tau_1) = \lim_{s \to 0} s\,\tilde{x}(s) \tag{4.17}$$

using the above result and (4.7) we can express the steady state lateral drift as:

$$\frac{\dot{z}_{ss}}{V} = \frac{B_o(\tau_o)\,\theta_{ss}}{(\mu_c K_A K_\alpha - \mu_\alpha)} \tag{4.18}$$

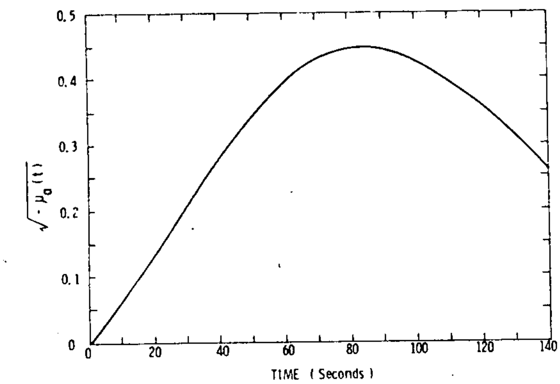

Fig. 7. Natural Clock For T-V Vehicle Model.

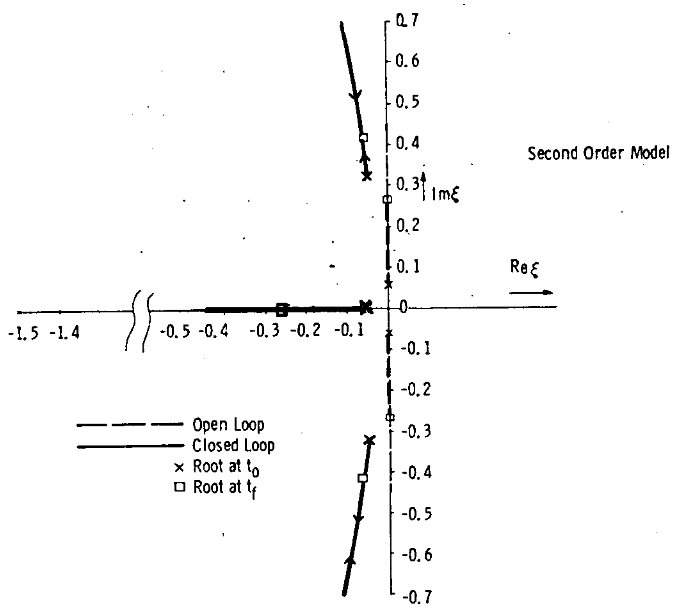

Fig. 8. Pole Configuration.

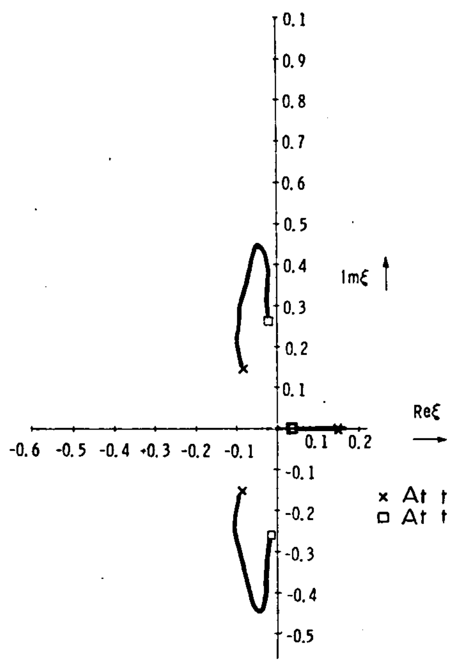

Fig. 9. Pole Configuration For Shuttle Boost (open loop).

where B_o is given in (4.10) and the coefficients vary on the τ_o scale. By similar reasoning θ_{ss} can be shown to be:

$$\theta_{ss} = \frac{-(\mu_c K_A K_\alpha - \mu_\alpha)}{\mu_c K_A (K_\theta + K_\alpha) - \mu_\alpha}\left(\frac{\dot{z}_{ss}}{V} + \alpha_w\right) \tag{4.19}$$

Therefore:

$$\frac{\ddot{z}_{ss}}{V} = -\frac{B_o(\tau_o)}{[\mu_c K_A (K_\theta + K_\alpha) - \mu_\alpha]}\left(\frac{\dot{z}_{ss}}{V} + \alpha_w\right) \tag{4.20}$$

$B_o(\tau_o)$ is a function of the feedback gains. If they are chosen such that $B_o \equiv 0$ then the lateral drift $\ddot{z}_{ss} = 0$ regardless of the wind input. This has now been shown to be continuously true throughout the time variation. This is the drift minimum condition[7], which can be written as:

$$K_A K_\theta (\mu_\alpha T_c + \mu_c L_\alpha) = (mg \cos \theta_o + T_T - D)(\mu_c K_A K_\alpha - \mu_\alpha) \tag{4.21}$$

This is valid continuously through the time variation. Time-varying feedback gains $K_A(\tau_o)$, $K_\theta(\tau_o)$, $K_\alpha(\tau_o)$ can be chosen to satisfy (4.21) which then results in a drift minimum condition. A number of choices are possible. For example, servo amplifier gain K_A and pitch attitude gain K_o being unity, $K_\alpha(\tau_o)$ is given by (4.21) for minimum drift. This is shown in Fig. 10. This, together with $K_R = 1$ results in a root configuration as in Fig. 11. It is seen that the system is now stable and responds more rapidly than the open loop system and has minimum drift. On the other hand, we can choose $K_\alpha = 0$ and determine $K_A(t)$ for minimum drift, with $K_\theta = K_R = 1$. The gain variation and the closed loop roots are now shown in Fig. 10, 12. It is seen that in both cases the lateral drift is minimum and the closed loop system has desirable stability and response characteristics. The response of the closed loop system is simply expressed as a damped oscillation with variable damping and frequency and has the form exp $(\tau_1(t))$ for each mode where $\tau_1(t)$ is a quadrature over the characteristic roots $\xi(\tau_o)$.

Conclusions

A method has been developed to synthesize the control of time-varying systems analytically. It is applicable to linear t-v systems with slowly varying coefficients. We have developed output-input relationship in the form of a transfer function for the t-v system, similar to time-invariant linear control theory. Further, the response of the system has been separated into fast and slow parts, by performing observations on different "clocks". For t-v systems the clocks are, of necessity, nonlinear functions—ie they count time at varying rates. In general the clocks have

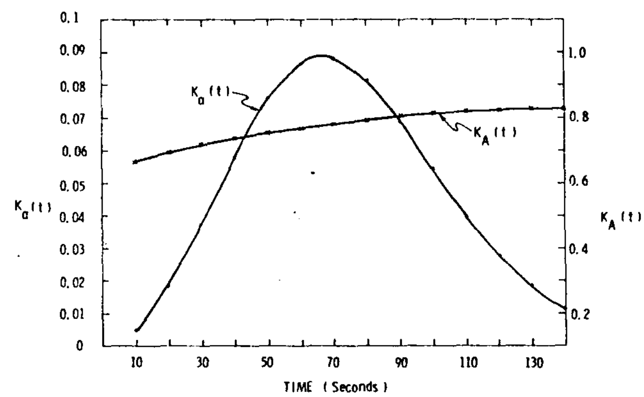

Fig . 10 Gain Profiles for Minimum Drift

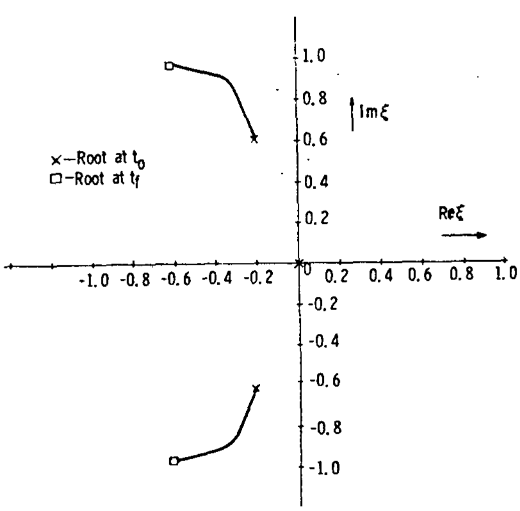

Fig. 11

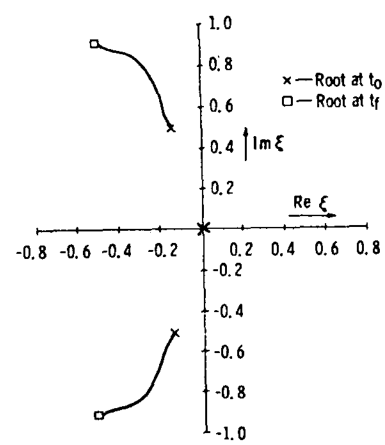

Fig. 12

Pole Variations for Minimum Drift (Closed Loop)

to be not only nonlinear functions but even complex quantities in order to describe oscillatory time-varying phenomena.

The advantages of the approach are evident. The "frozen" approximation to a t-v system is established on a rigorous mathematical basis in the light of the multiple scales theory. Such a representation can be obtained by considering purely linear clocks. Indeed, the frozen representation, which is valid only for short times around the instant of freezing, can now be replaced by a more accurate representation by our theory, which is valid throughout the time variation and not just at specific instants of time. Further, the case of constant coefficients arises as a natural limit, being a special case of our general result. Stability of the t-v system can be determined easily by our asymptotic theory. As we are dealing with analytical representations which are simply calculable, we can employ them in different control theoretic tasks such as control synthesis, parameter sensitivity studies, closed loop control and stability analysis.

The method has been illustrated by applying it to the analysis of the shuttle vehicle dynamics during boost phase and to the design of a control system. Analytical representations of some parameter variations are derived on a least squares basis and the condition of gain variations for minimum drift are derived. Control system is designed in the transformed space for the second order model representing the vehicle at high dynamic pressure. For the full third order model, control gain variations are derived for two control configurations using the minimum drift condition in the transformed ξ space. It is important to note that the drift minimum condition, derived rigorously, is valid throughout the time variation. Solutions in real time t, obtained after inverse Laplace transformation and restriction, are expressed as exponentials of quadratures over $\xi(t)$. Control systems for t-v systems can now be designed on this basis because of the analytical representation of the response of general time-varying systems.

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